System Identification for Nonlinear Aerodynamic Flight Regimes

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A new development for the identification of aerodynamic forces and moments in nonlinear regimes is presented, with applications to simulated and flight test response data. This development is a two-step method. The first step is the application of an algorithm which determines the order and coefficients of polynomial expansions of the nonlinear aerodynamic forces and moments which characterize the nonlinear flight regime. The second step is the use of a nonlinear six-degree-of-freedom maximum-likelihood algorithm, which accurately estimates the values of the polynomial coefficients. This method has been applied to simulated and flight test data for a twin engine swept wing fighter aircraft in stall/post-stall aerodynamic regimes.

I. Introduction

A RECURRING problem for flight test data analysis is the estimation of nonlinear aerodynamic coefficients. Such estimates are essential for quantitative approaches to handling qualities evaluation, aerodynamic modeling for real-time simulations, and verification of analytical and wind-tunnel predictions of aerodynamic parameters.

System identification is a systematic methodology for exploiting test data to provide such accurate coefficient estimates. Its application to multivariable nonlinear systems, however, requires an additional step before actual identification of parameters is attempted. This step is a mathematical model determination stage where, of all possible nonlinear aerodynamic effects postulated, those of greatest significance are isolated and retained for further analysis. There are two basic reasons for this reduction. First, a complete nonlinear model places severe computational demands on parameter identification algorithms. Hence the model generally must be reduced in order. Secondly, if all possible aerodynamic contributions are retained, the problem is "overparameterized" by the inclusion of terms that are not identifiable. This may produce divergence in the identification algorithm.

This paper presents a method for performing the system identification task to the important nonlinear flight regime in the stall/post-stall angle of attack range. The method consists of a model-determination algorithm for estimating which nonlinear parameters significantly affect aircraft responses and a nonlinear, six-degree-of-freedom maximum-likelihood algorithm for accurately identifying the values of the selected nonlinear terms (Sec. II). It is applied to simulated data (Sec. III) and flight test recorded responses (Sec. IV) of a twin engine, swept-wing, high-performance fighter aircraft. General conclusions regarding estimation of aerodynamic parameters in nonlinear regimes are presented in Sec. V.

II. The Integrated Parameter Identification Process

Requirements for Identification

Aircraft parameter identification is the process of extracting numerical values for the aerodynamic stability and control coefficients from a set of flight test data (e.g., a time history of the flight control inputs and the resulting aircraft response variables). The complete procedure involves three steps (Fig. 1): a) model and parameter selection, b) parameter estimation, and c) model vertification. The fundamental complexity of these three elements requires a strong reliance on information available from wind-tunnel data and knowledge of aircraft physics. The particular value of such prior information is the manner in which it can be used to formulate the *possible types of forces and moments*. The isolation of the *specific nonlinear effects* in post flight analysis is the model determination phase. The parameter identification is the quantification of the specific aerodynamic terms selected by the model-determination phase. Model verification is the application of various techniques to increase confidence that the identified model adequately represents the vehicle responses. Such techniques include the calculation of the confidence levels of the estimates, consistency with results of previous wind-tunnel and flight tests, and the ability of the identified model to predict responses of the vehicle with different inputs.

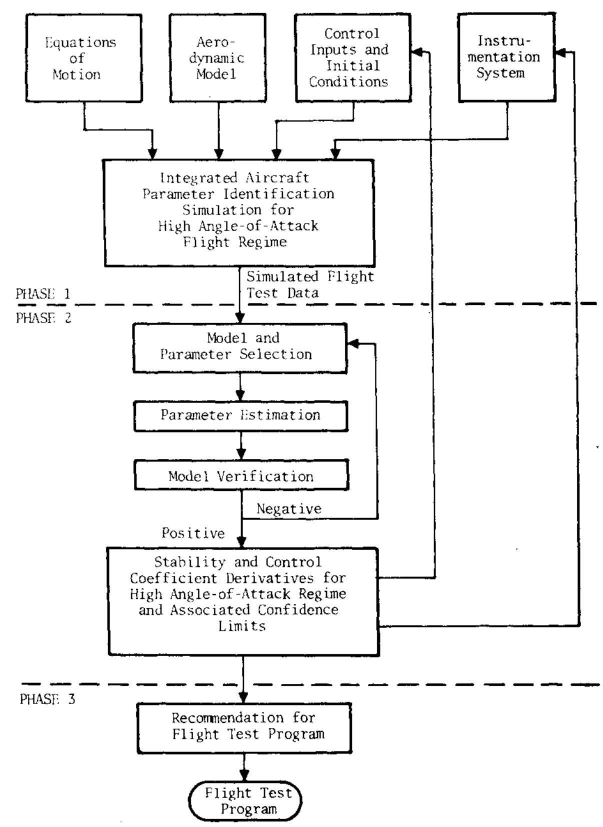

Fig. 1 Implementation of the design of an integrated parameter identification process for high-angle-of-attack application.

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Techniques for Simulation, Model Determination, and Parameter Identification

In this section, we review the techniques that have been developed for implementation of the integrated parameter identification process.

Simulation

The simulation objective is to provide a controlled data base for validating the estimation procedures and for isolating potential identification problems that might arise from actual flight test data. Requirements for inputs and instrumentation, for example, may be specified from such simulations prior to the flight tests. Such requirements may be difficult to individually detect from any specific set of flight data due to the complex response and data interactions that exist in any actual system. The simulation described in Ref. 1 was designed as a generator of data characteristic of that from the responses of high-performance aircraft.[4-6] The simulation was subsequently compared with reported results of the stall/spin tests of a swept wing fighter aircraft.[3] Responses such as pitch-up, yaw departure, and spin were obtained with the simulation. In addition, it was found that wing rock would be simulated, and that the roll, sideslip, aileron deflection amplitudes, and angle-of-attack occurrence were comparable with that recorded in the flight test.

As shown in Refs. 1 and 2, the simulation used to generate data is an extensive and detailed representation of the aircraft and data error sources. Both the dynamic equations and the measurements are nonlinear. The simulation aspect of the process reported here is unique in the application of identification algorithms to simulated data. Typical approaches assume a polynomial form for nonlinear parameter variation, generate the time-history data with this model, and then attempt to reconstruct the polynomial from the data.[8-10] Such an approach is certainly useful for testing a program, but it may tend to place more confidence in the effectiveness of an algorithm than is justified. This follows because identification of the same functional form that generates the data disregards the modeling error, which may occur by approximating the actual unknown function by an assumed or *a priori* function. The approach used in this work is to generate that data by an aerodynamic model that is of higher order than the model identified (e.g., data is in "table format"). Hence, the integrated parameter identification process of Fig. 1 contains the element of modeling error effects explicity.

Model Structure Determination

Having determined an analytical, *a priori* form for the aerodynamic forces and moments and selected the principal axis systems, the framework is established for estimating the structure and parameters appropriate to a given data. Polynomials are chosen as the basis of the identification model for this work. This is itself an assumption about the physics of the aircraft aerodynamics. The assumption is historically based on the dependence between force and moment coefficients and independent variables (α or β, for example) that is observed in wind tunnels. Recent work in England[11] has demonstrated the validity of such approximations with actual aircraft responses in the subsonic regime. The general expansion for any specific force or moment coefficient is

$$C(\bar{\alpha}+\alpha,\bar{\beta}+\beta)=C_0(\bar{\alpha},\bar{\beta})+\sum_i C_{\alpha^{(i)}}\alpha^i+\sum_i C_{\beta^{(j)}}\beta^j + \sum_i\sum_j C_{\alpha^{(i)}\beta^{(j)}}\alpha^i\beta^j \tag{1}$$

where $\bar{\alpha}$ and $\bar{\beta}$ are reference angle of attack and sideslip angle, respectively.The specific coefficients for which such an expansion is used are listed in Table 1. It should be noted that the method may be used with piecewise polynomials (e.g., splines) or spectral functions without extensive modification.

Wind-tunnel test results (and any other information about the physics of the maneuvers) are used to define all the possibilities of the polynomial functions for the forces and moments. Then, the actual response data obtained is used to specify which of these functions are most probable by a subset regression. The goal is to identify only those polynomial coefficients that are required to reproduce the actual force or moment characteristics.

The subset regression is a sequential least squares algorithm that adds and deletes variables to a particular model in an iterative manner, isolating a significant subset of the possible polynomial coefficients. The algorithm uses statistical hypothesis testing techniques based on the Fisher *F*-ratio (see the Appendix). Formally, this ratio measures the difference in fit error with the current model relative to the error due to noise and model uncertainties. In the regression approach for model structure, *F*-ratio plays a central role. It is the ratio "fit goodness to fit error" weighted by the degree of freedom. A "total" *F*-ratio measures the entire model fit relative to the error and a "partial" *F*-ratio measures the incremental improvement in fit due to addition or deletion of a parameter in the model. A generalized flow chart is shown in Fig. 2. Starting with a list of possible variables, the algorithm enters the first variable with the highest partial correlation to the observations y. The contribution of this variable to reducing the fit error is made and a new variable entered. Subsequent tests add and delete variables to improve the "fit." The final subset of parameters, θ, which results from the procedure is one within confidence bounds set by the user (say, 95% or 99%).

That the *F*-ratio is a measure of the "optimal number" of parameters is shown in Fig. 3, corresponding to an example to be discussed in more detail in Sec. III. This figure illustrates the need to evaluate model suitability on a criterion other than fit error which is proportional to the multiple correlation coefficient, R^2. As shown, R^2 is a monotonically increasing funciton of the number of parameters. The *F*-ratio, however, has a maximum with seven parameters (obtained after deleting one variable of small significance). In general, it has been found that the subset *F*-ratio will have local maxima, beyond the first, as parameters are added. A general criterion used for selection parameters to be identified is to delete all parameters included in the regression past the global maximum.

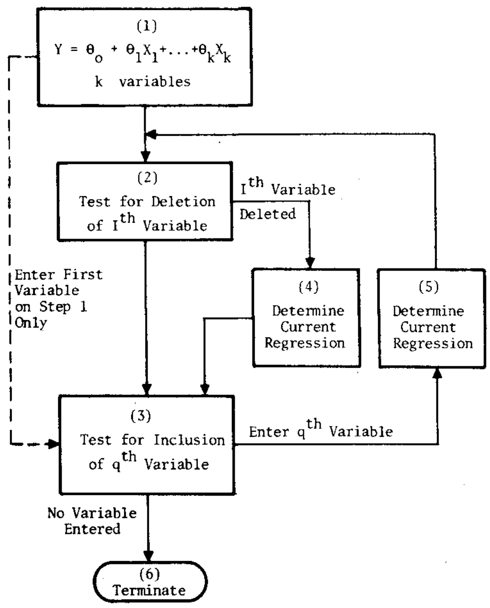

Fig. 2 Generalized flow chart of the subset regression algorithm.

Table 1 Coefficients expanded for high-angle-of-attack aerodynamic models

EQUATION	STATIC COEFFICIENT	CONTROL EFFECTIVENESS	DYNAMIC COEFFICIENT
u	$C_x(\alpha,\beta)$	$C_{x_{\delta_s}}$	C_{x_q}
v	$C_y(\alpha,\beta)$	$C_{y_{\delta_a}}$ $C_{y_{\delta_r}}$	C_{y_p}
w	$C_z(\alpha,\beta)$	$C_{z_{\delta_s}}$	C_{z_q}
p	$C_\ell(\alpha,\beta)$	$C_{\ell_{\delta_a}}$ $C_{\ell_{\delta_r}}$	C_{ℓ_p} C_{ℓ_r}
q	$C_m(\alpha,\beta)$	$C_{m_{\delta_a}}$	C_{m_q}
r	$C_n(\alpha,\beta)$	$C_{n_{\delta_a}}$ $C_{n_{\delta_r}}$	C_{n_p} C_{n_r}

The algorithm not only identifies the most significant parameters, but also finds least square estimates of their true values. In general, these estimates will be in error (biased due to measurement noise and high-order modeling errors). The algorithm may require numerical differentiation of measurement time histories to provide estimates of states for the regression. Numerical simulation of the effect of noise on such differentiation was conducted. Such differentiation was found to have little effect on the parameter subset below the maximum F-ratio,[1] although the parameter estimates themselves became biased. As such, they can be used for start-up values of the maximum-likelihood algorithm to reduce computation time and improve convergence but may not be considered as final estimates themselves.

Primary emphasis is placed on the static coefficient structure for the modeling tasks here. All static terms could minimally be expressed as linear combinations of $(C_i\alpha + C_j\beta)^n$, where $n = 1,2,3$. In addition, the coefficients of C_m could be expanded to α^9 and C_n to β^5. Control effectiveness coefficients and dynamic coefficients were generally limited to first-order expansions in α only.

Parameter Identification Procedure

The extraction of aerodynamic derivatives from flight data has received considerable attention during the last three decades, and the most recent efforts are given in several references.[12,14] With the availability of fast digital-computing machines and efficient computation algorithms, many powerful digital methods have been developed. Examples of digital methods are various equation error methods,[15-16] output error methods,[17,19] Kalman filter/smoother approach[9] and the maximum-likelihood technique.[2,8,20-22]

The maximum-likelihood techniques is most general (Fig. 4). By assuming unknown elements of process noise and measurement noise covariances and other instrumentation errors as parameters, it considers these noise sources and also estimates them, if not known *a priori*. The method determines parameter values, which maximize the likelihood function of the parameters given the measurements and *a priori* information. The likelihood function has the same form as the conditional probability of the observations given the parameters. It is customary to work with the logarithm of the likelihood function. The method, as applied to parameter identification in nonlinear dynamic systems with measurement and process noise is a combination of two steps: 1) Kalman filter for state and its covariance, 2) Gauss-Newton method for parameter estimates, and associated covariances, also unknown noise statistics.

Figure 4 is a flow chart of the procedure steps. The details of the above steps are given in Ref. 2. In addition to estimating the parameters in the state and measurement equations, the procedure also determines the covariance of

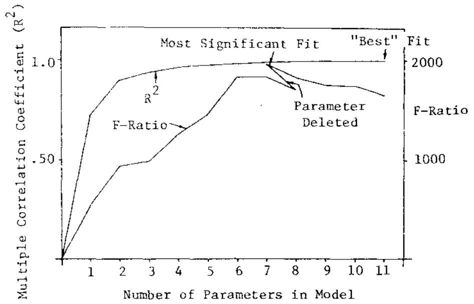

Fig. 3 Multiple correlation coefficient (R^2) and F-ratio variation as parameters are added to model (lateral case).

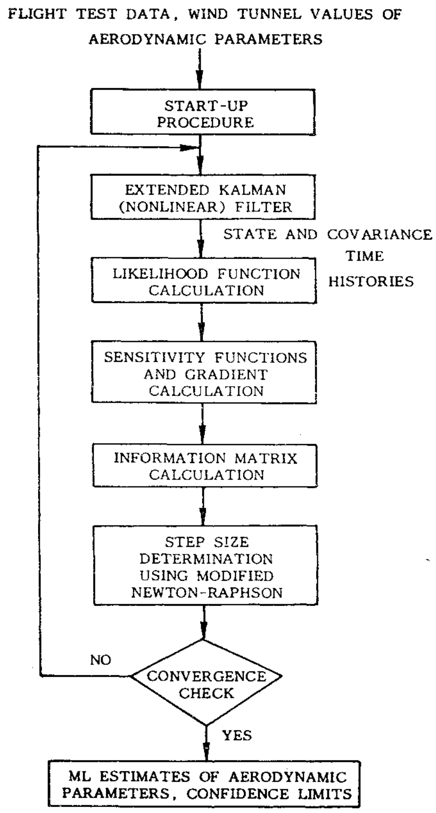

Fig. 4 Flow chart of maximum-likelihood identification program.

errors in parameter estimates. If the model, whose parameters are identified, is a true representation of the system in the region of operation and the sampling rate is high, the maximum-likelihood method gives unbiased estimates of parameters for long data records. With increasing amounts of data, the estimates converge to their true value almost certainly. It can be shown that the technique extracts all information about the parameters from data; in other words, the method is efficient.

Input Design

The model determination and parameter identification aspects of the integrated parameter-identification procedure requires careful design of flight tests; in particular, it is necessary to make a good choice of inputs and the instrumentation.

Many methods have been suggested to design inputs that would give good identifiability of stability and control coefficients over a range of states in nonlinear systems. One way could be to carry out flight tests around many angles-of-attack and sideslip angles covering the range of interest. Inputs based on linear models are designed at each of these points.[23,24] This would given an accurate description of each aerodynamic derivative, but may be practically infeasible because it would require excessive flight testing time and data-processing time. In addition, it may require carrying out flight tests in regions where the time conditions cannot be reached or the airplane is unstable and/or unsafe.

An iterative technique is useful in the design of input signals for identifying parameters in complicated systems. As a first step, analytic inputs are designed based on simplified models. State-time histories are generated and inputs are evaluated based on a more accurate but complex model. This gives information about poorly identifiable directions in the parameter space and about poorly excited modes. In the implementation used here, the subset regression program is used to determine identifiable directions resulting from a certain input. A knowledge of the deficiencies in the chosen input, together with known systems behavior, is used to modify the input. The process is repeated until an acceptable input is obtained. A similar procedure is used for selecting the instruments.[7]

Identification in the Stall/Post-Stall High-Angle-of-Attack Regime

In the following two sections evaluation of the integrated parameter identification process is carried out using simulated and actual aircraft flight-test data for the twin-engine swept-wing fighter. State and measurement model equations used in this maximum-likelihood procedure are detailed in Ref. 2. Nine state equations and eight measurement equations containing thirty-three nonlinear, aerodynamic coefficients compose the identification model (each of these coefficients are further expanded).

In the identification procedure, the subset regression program is used on the simulated flight test data to identify the model structure and give initial estimates of the selected aerodynamic coefficient polynomial expansion parameters. The maximum-likelihood program is then used to refine the parameter estimates obtained from the regression analysis to yield the final parameter estimates and a measure of confidence associated with those estimates.

In addition to evaluating the identification procedure on a variety of simulated-flight test experiments, investigations are also carried out to assess the effects of different levels of measurement noise, process, noise, control input variations, identified parameter set size, and data length.

The identifiability of the stall/post-stall regime is the prime objective of the applications conducted for the identification process. This angle-of-attack regime (10° to 25° and beyond) is characterized by multiple nonlinearities in the pitch, roll, and yaw moments and forces. The central objective of this application is that resolution of the identifiability problems for this range is the prime requisite for any high-angle-of-attack identification procedure.

III. Application to Simulated Data

The preceding sections have presented an overview of the integrated parameter-identification process. In this section, we discuss the application of this process to simulated data from the detailed six-degree-of-freedom nonlinear simulation of an advanced twin-engine, swept-wing fighter aircraft.

Lateral Results

In the baseline lateral case, control inputs are chosen to excite the lateral-direction modes of the aircraft. Applied controls are a half-period sine-wave pulse in rudder δ_r of 10° amplitude and 2.5-sec duration followed by a full-period sine-wave doublet in ailerons of 7° amplitude and the same duration. Total data length is 10 sec real time, sampled every 0.1 sec to give 100 points for each measured variable. Initial flight conditions are $\alpha = 17.5°$, $\beta = 0°$, and all rates zero. The baseline runs are corrupted by measurement noise.[2] Twelve parameters are selected by the subset regression program for this baseline case. Figure 5 shows typical time history fits of various aerodynamic coefficients showing actual and estimated values with $\pm 2\sigma$ confidence limits. The most prominent characteristic of both demonstration cases is the excellent measurement estimate fits to the actual data. Estimated 2σ confidence limits bracket the measurement noise-induced variations of the actual data.

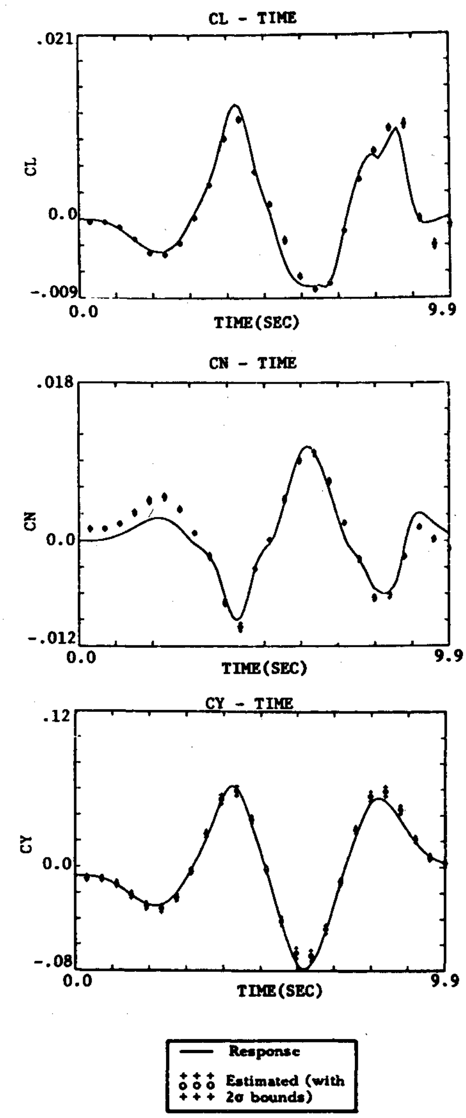

Fig. 5 Lateral static coefficient response (simulated or estimated) to combined aileron/rudder input.

Estimate fits to the nonlinear aerodynamic coefficients reveal a range from excellent estimates to fairly significant biased estimates. For the lateral motion case, the major coefficient estimates, C_l, C_n, and C_y are very good; again, with confidence limits bracketing actual data most of the time. Estimates of control derivatives such as $C_{n_{\delta_r}}$, $C_{n_{\delta_a}}$ and $C_{l_{\delta_a}}$ tend to be biased, however. The two prime reasons for the biased estimates are 1) modeling errors due to higher order variation of actual coefficients with α and β than allowed for in coefficient model polynomial expansion; 2) the low amount of information about control derivatives available as a result of short time of control application. Significant improvements in control effectiveness of parameter estimates are achievable by modifying the inputs and increasing the data length.[2]

Longitudinal Results

The longitudinal mode with inputs to excite pitch rate, vertical velocity, and longitudinal velocity is a fundamental aspect of this effort. The stabilator (elevator) input for the

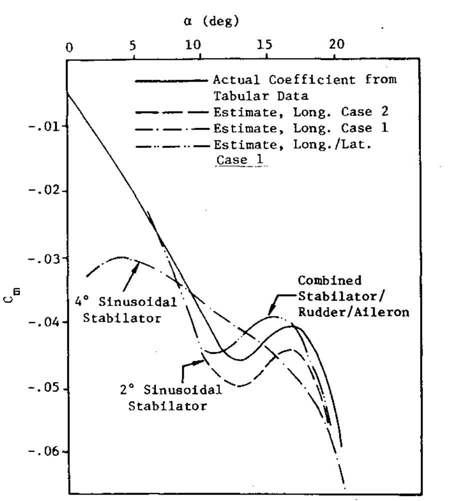

Fig. 6 Estimates of C_m vs α for different inputs.

baseline simulated longitudinal case is a continuous sine wave superimposed on the initial, constant stabilator angle. With a period of 4 sec, the input is designed to be approximately at the short period mode frequency of the aircraft and, therefore, yields the most information about the parameters governing that mode. Amplitude of the input is 4°. Again, the data length is 10 sec real time, and there are 100 sample points. A second case is also a sinusoidal stabilator input, but with smaller amplitude of 2°. A third coupled response was generated in which longitudinal and lateral inputs were simultaneously applied. A total of 31 parameters were identified for this latter case.

Results for these three inputs are shown in Fig. 6. Whereas the 4° stabilator oscillation is very poor at the limits of the best angle-of-attack range, the 2° stabilator and coupled-inputs are much better.

One objective of parameter identification techniques is the ability to use model parameter values estimated from one set of data to predict the responses of a different set of data. Of course, the operating range of the data to be predicted should not extend beyond the valid limits of the parameter estimates, but within that constraint, the predicted response should closely approximate the measured response. For the model to be of engineering utility, it must not only be able to reproduce the data upon which it is based but also be able to closely match response to different inputs. Figure 7 shows the predicted and actual response of the aircraft using parameter estimates from previous lateral/longitudinal coupled response estimates. The inputs for this case, however, are considerably different in form and amplitude from those used to generate previous estimates.

The measurement estimate fits for the prediction indicate that frequency and damping are accurately specified. A bias is present in yaw rate and roll rate, but it is a small effect (note scale of yaw and roll rates). It is assumed from this relatively simple 31 parameter estimate model that it is successful in faithfully reproducing the far more complex simulation of real aircraft flight characteristics.

IV. Flight Test Data

High-Angle-of-Attack Longitudinal Flight Data for a Swept-Wing Fighter

The high-angle-of-attack results reported here are for flights conducted by the U.S. Air Force[3] on the subject fighter aircraft at 40,000-ft altitude. Only Record 14 from Flight Test 165[3] is used for the extraction of longitudinal and lateral stability and control coefficients at high angles of attack. In this record, the elevator was used to increase the angle of attack steadily from about 15° to over 40° over a 20-sec period. The airplane stalls at about 25° and finally the unstable lateral motions produce a rolling departure. The thrust of the engines during this maneuver is assumed constant. There are measurements of linear accelerations, angles, and angular rates.

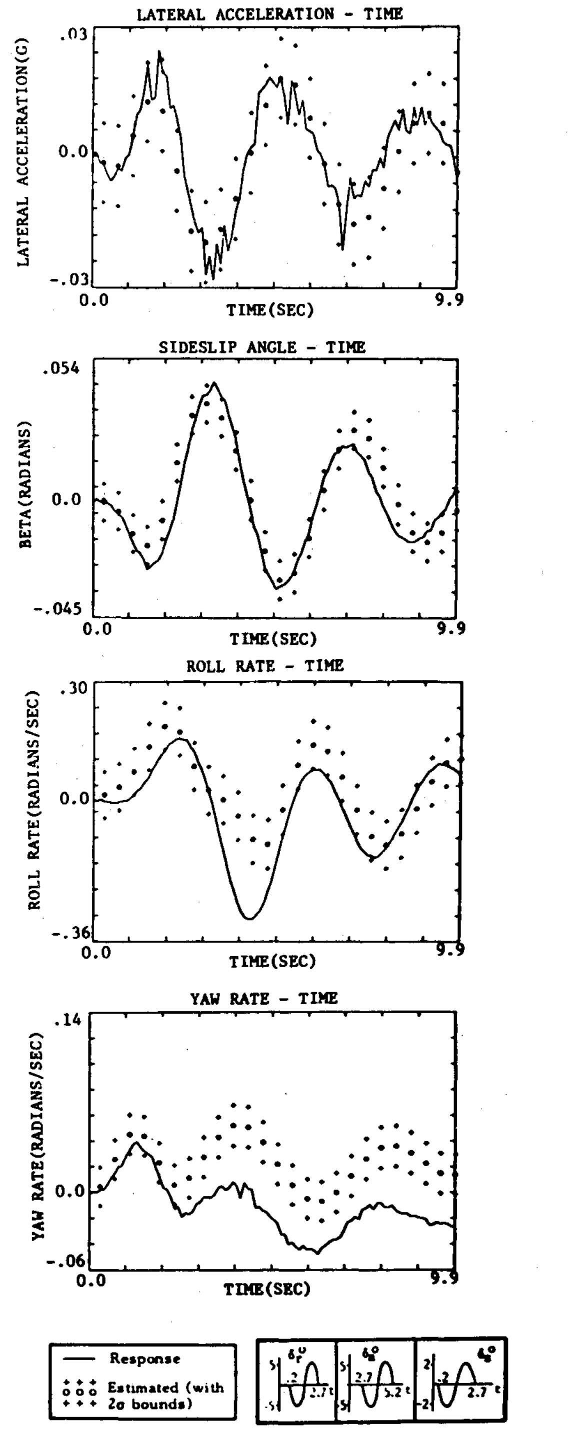

Fig. 7 Lateral predicted and simulated response to a combined stabilator/aileron/rudder input.

First, the subset regression is used to determine a set of relevant aerodynamic derivatives that define the model structure. The angular rates were differentiated numercially to obtain angular accelerations required for the subset regression. As explained previously, the aerodynamic derivatives are expanded only as selected functions of angle of attack, α, and sideslip angle, β. The set of functional variables allowed for each moment and force coefficient was discussed in Sec. II.

Adequate expansions of forces and moments for the longitudinal motions from the subset regression program are

$$C_x = C_{x_0} + C_{x_{\alpha^2}}\alpha^2 + C_{x_{\alpha^4}}\alpha^4 + C_{x_\beta}\beta + C_{x_q}q$$

$$C_z = C_{z_0} + C_{z_\alpha}\alpha + C_{z_{\alpha^2}}\alpha^2 + C_{z_{\beta^2}}\beta^2 + C_{z_q}q + C_{z_{\delta_s}}\delta_s + C_{z_{\alpha\delta_s}}\alpha\delta_s$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\alpha^2}}\alpha^2 + C_{z_{\beta^2}}\beta^2 + C_{z_q}q + C_{z_{\delta_s}}\delta_s + C_{z_{\alpha\delta_s}}\alpha\delta_s$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\alpha^2}}\alpha^2 + C_{m_{\alpha^3}}\alpha^3 + C_{m_\beta}\beta + C_{m_q}q + C_{m_{\delta_s}}\delta_s$$

There are several important things about these expansions. The order of expansion in terms of angle of attack is different for each coefficient. Also, in the expansion for C_x, the fourth-order expansion term must be included, whereas, the third-order expansion term is not. There is a significant coupling in the longitudinal motions from variations in the sideslip angle. The expansion of the control coefficient is different for the three moment and force coefficients. The model representation selected by the subset regression program is used with the maximum-likelihood approach for the estimation of parameters. The starting values of the parameters are taken from the results of the regression. A comparison of the time history plots of the measurements of angle of attack, pitch rate, fore-aft acceleration, and vertical acceleration with the predicted values of the measurements based upon the identified parameter values are given in Ref. 2. The identified and the wind-tunnel values of C_m as a function of angle of attack, at zero sideslip angle, are also shown in Ref. 2.

High-Angle-of-Attack Lateral Flight Data for a Swept-Wing Fighter

The first 15 sec of data from the selected model for the aircraft lateral directional motions are used. There are nonlinear terms in the model, the lateral force and moment coefficients being adequately described by the following equations.

$$C_\ell = C_{\ell_0} + C_{\ell_\beta}\beta + C_{\ell_{\alpha^2\beta}}\alpha^2\beta + C_{\ell_r}r$$

$$C_n = C_{n_0} + C_{n_{\alpha\beta}}\alpha\beta + C_{n_p}p$$

$$C_y = C_{y_0} + C_{y_{\beta^3}}\beta^3 + C_{y_{\alpha\beta}}\alpha\beta + C_{y_{\alpha\delta r}}\alpha\delta r + C_{y_{\alpha p}}\alpha p$$

Notice that at the high angle of attack, the lateral controls are almost ineffective. The lateral motion can be adequately explained by an unstable system driven by its coupling with the longitudinal motions.

The time history plots of the measured outputs and predicted outputs are compared in Fig. 8. The comparison of the identified values and the wind-tunnel values[6] of coefficients C_{y_β} and C_{n_β} is shown in Fig. 9 over the range of angle of attack encounted in the flight test. The identified variation of C_{y_β} with α, $C_{y_{\alpha\beta}}$ agrees with the slope of the wind-tunnel value, but appears to be biased. This derivative is difficult to identify from this response because of the small lateral accelerations that were recorded. C_n is important because it contributes to the lateral stability of the aircraft at high angles of attack, and agrees well with the wind-tunnel value.

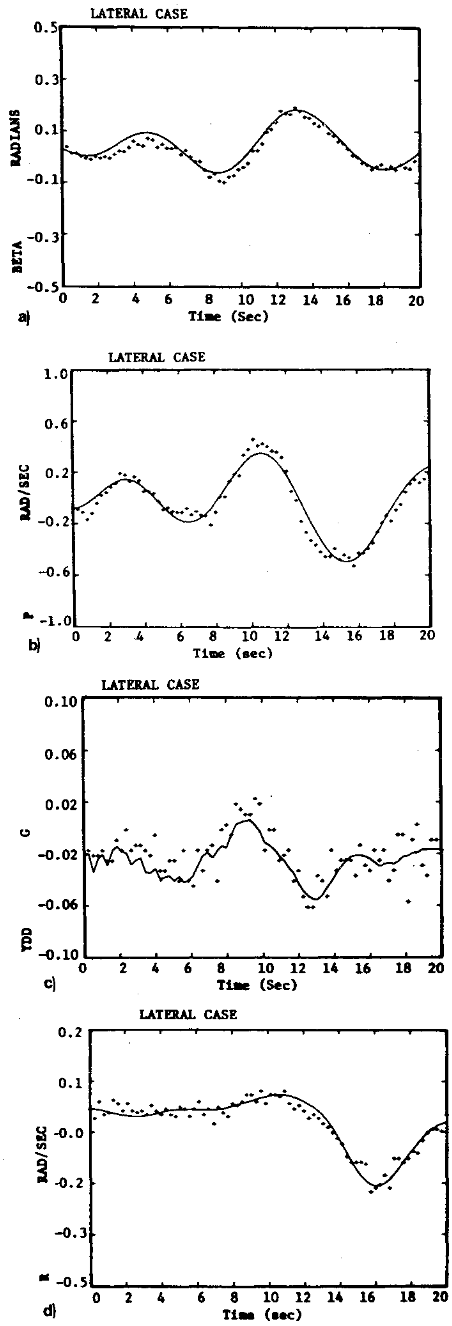

Fig. 8 a) Measured sideslip response compared to response of identified model. b) Measured lateral acceleration response of identified mode. c) Measured roll rate response of identified model. d) Measured yaw rate response of identified model.

V. Summary and Conclusion

The Integrated Parameter Identification Procedure

The basic objective of the integrated parameter identification procedure is to most fully exploit the theoretical and computational versatility of the maximum likelihood method to yield a practical data processing tool. The likelihood function contains all information about known parameters, if the model is correct. Primary emphasis has thus been placed on specifying the best possible model estimate. This specification is achieved with an algorithm based on subset regression.

Application of this model determination program demonstrates a significant improvement in maximum-likelihood efficiency, both in required computation and in accuracy of results. The improvement in performance of the entire process is based on the following characteristics of the regression method:

1) The subset regression method selects parameters on their ability to match the measured response. By selecting the optimum number of variables of accomplish this match, the most significant variables are isolated.

2) The method yields *a priori* estimates from the data above, and does not itself require initial estimates. Although usually biased (due to noise and numercial differentiation), the estimates given by the program are frequently better than *a priori* estimates from other sources.

3) The method gives significant evaluations on the estimated parameters which serves an essential role in final

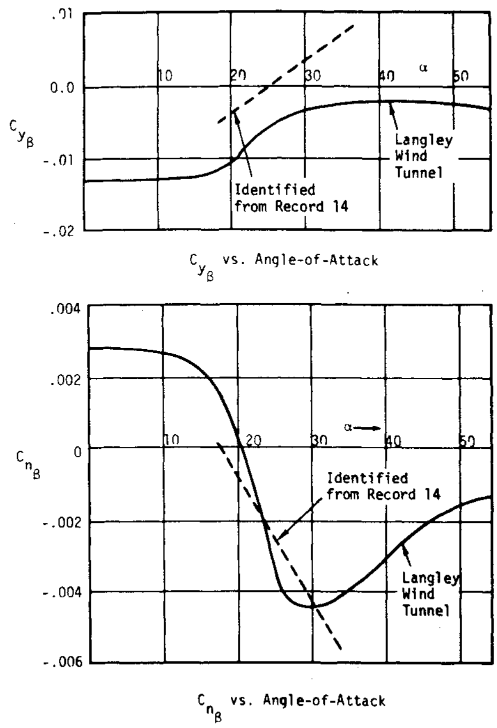

Fig. 9 Identified coefficients from lateral response.

selection of a model structure to be used in the maximum-likelihood algorithm.

4) The computational requirements of the regression algorithm are modest, and the algorithm can be used quickly not only to evaluate input designs, but also effects of measurements errors.

The implementation of the maximum likelihood for this work can accept either linear or nonlinear aerodynamic expansions of the force and moment coefficients. These expansions can be Taylor series expansions, spline functions, or other appropriate representations. Furthermore, this algorithm can process parameters for only one dynamic equation or all six. The program includes calculations of parameters significance level.

Application to Simulated and Flight Test Data for the High-Angle-of-Attack Stall/Post-Stall Regime

The evaluation of the integrated parameter identification process has been conducted with simulation and flight data. The simulation is highly nonlinear and of much higher order than the identification models used.

Primary emphasis has been placed on the identification of static force and moment coefficients. Greatest errors have been found in estimates of the dynamic rate derivatives and control effectiveness derivatives. These errors are attributed to the relatively low response rates which were obtained for inputs to identify the important static derivatives. Inputs may be modified using input design algorithms discussed here to improve the estimation of these derivatives. It must be noted that the errors of the dynamic derivatives that were obtained could not be found by comparing the *history matches of the measurement of rate,* as these were excellent in all cases. Only with comparison of the actual *parameter time histories* were these errors observed.

Techniques for improving parameter estimates include input design and increased data length (where possible). Such modifications have a significant effect on flight test planning and should be available for identification of important but difficult to excite parameters. The integrated parameter identification process as developed in this work is amenable to such flight test design, using procedures detailed in Ref. 23.

The confidence established in the procedure led to evaluation of the identification results by a prediction criterion. Specifically, the parameter estimates from a specific lateral maneuver with one input are used to predict the response of the simulation to another input (for roughly the same flight regime). This prediction capability was verified with good results.

It is concluded that the application of the integrated parameter identification process developed for nonlinear flight regimes offers significant improvements in the ability to identify not only parameters but also the entire system structure and parameters.

Appendix: Model Structure Determination by Subset Regression

The subset regression technique (also called stepwise regression) has been used in statistics to determine a set of independent variables that determine the value of the dependent variable to a specified accuracy. The same technique can be used to determine the model structure of a nonlinear dynamic system, in particular the high-angle-of-attack problem. With measurements of accelerations and state variables, the equations of motion can be written as

$$y = X\theta + \epsilon \tag{A1}$$

where y is an $m \times l$ vector of accelerations, X is an $m \times p$ matrix of state variables and nonlinear functions of state variables, θ is a $p \times l$ vector of parameters and ϵ is the residual. The set of important parameters is determined by performing a correlation analysis between y and x. The parameters are included in the regression equation one at a time until the entire model is determined (see Ref. 26 for details).

At any point in the analysis the regression equation $y = X\theta + \epsilon$ can be partitioned as

$$y = X_l\theta_l + X_2\theta_2 + \epsilon \tag{A2}$$

where X_l includes q variables and X_2 contains p-q variables. Then

$$y - X_l\theta_l = X_2\theta_2 + \epsilon \tag{A3}$$

which shows that an estimate of θ_2 could be obtained by regressing the residuals from the regression of y on X_l (i.e., which estimates θ_l). Then the vector $y - x_l\theta_l$ is regarded as a new observation, say y^l, which may be regressed on X_2 to estimate θ_2. This decomposition can be applied to each possible subset of variables, X_i, "bringing in" new variables from the right to left hand side of Eq. (A.3). The requirement on bringing in new variables may be satisfied by examining the significance of each variable.

The F-test may be used to determine the significance of a single parameter by noting the estimate of the variance σ^2, s^2, is distributed as $\sigma^2 X^2_{m-p}$. Hence, $s^2/\sigma^2 \sim (X^2_{m-p})/(m-p)$. Then, for the parameter θ_i

$$\frac{\hat{\theta}_i - \theta_i}{s_{\theta_i}} = \frac{(\hat{\theta}_i - \theta_i)/\sigma_{\theta_i}}{s_{\theta_i}/\sigma_{\theta_i}} \tag{A4}$$

where s_{θ_i}, is the standard error of θ_i, which is

$$S_{\theta_i} = s\sqrt{s_{ii}} \tag{A5}$$

where s_{ii} is the square root of the ith diagonal term of $(X^T X)^{-1}$.

Since $(\hat{\theta}_i - \theta_i)/\sigma_i \sim \eta(0,1)$, it follows that, by definition of Student's t-distribution that

$$\frac{\hat{\theta}_i - \theta_i}{s_i} \sim t_{m-p} \tag{A6}$$

In particular, it is desired to test the hypothesis $\theta_i = 0$ (i.e., y does not depend on θ); the static $t = \theta_i / s_{\theta_i}$ is used. It is shown in Ref. 26 that the F-distribution with 1 and $(m-p)$ degrees of freedom is equivalent to the t^2 distribution with $m-p$ degrees of freedom. Hence, the significance of individual regression coefficients, θ, is determined from F-ratios

$$F = \theta_i^2 / s_i^2 \tag{A7}$$

If the ratio (A.7) indicates a variable is not significant, then the variable is deleted. To bring in another variable, the partial correlation coefficients of all other parameters are examined. To form the F-ratio for these coefficients

$$r^2_{\hat{y}\tilde{x}_j} = \frac{(\theta_j / s_{\theta_j})}{(\theta_j / s_{\theta_i})^2 + (m-q)} \tag{A8}$$

where q is the number of variables already in the regression. The corresponding F-test is

$$F_j = \frac{r^2_{\hat{y}\tilde{v}_j}\,(m-q)}{(1 - r^2_{\hat{y}\tilde{x}_j})} \tag{A9}$$

The variable (F-ratio with 1 and $m-q$ degrees of freedom) is calculated for each of the remaining variables. The variable with the highest value is then brought into regression. This process is repeated until all relevant parameters are included in the regression.

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References

[1]Hall Jr., W.E., "Identification of Aircraft Stability and Control Derivatives for the High Angle-of-Attack Regime," Tech. Rept. No. 1, prepared for the office of Naval Research under Contract N00014-72-C-0328, June 1973.

[2]Hall Jr., W.E., Gupta, N.K., and Smith, R.G., "Identification of Aircraft Stability and Control Derivatives for the High Angle-of-Attack Regime," Tech. Rept. No. 2., prepared for the Office of Naval Research under Contract N00014-72-C-0328, 1974.

[3]McElroy, C.E. and Sharp, P.S., "Stall/Near Stall Investigation of the F-4E Aircraft," Air Force Flight Test Center, Edwards Air Force Base, FIC-SD070-20, Oct. 1970.

[4]Etkin, E., *Dynamics of Atmospheric Flight*, Wiley, New York, 1971 (also *Dynamics of Flight*, Wiley, New York 1959).

[5]Grafton, S.C. and Libbey, C.E., "Dynamic Stability Derivatives of a Twin Engine Jet Fighter for Angles-of-Attack from -10° to 110° and Sideslip Angles from -40° to 40°," NASA TND-6091, Aug. 1971.

[6]Anglin, E.L., "Static Force Tests of a Model of a Twin Engine Jet Fighter Airplane for Angles-of-Attack from -10° to 110° and Sideslip Angles from -40° to 40°," NASA TND-6091, Aug. 1970

[7]Sorensen, J.A., "Analysis of Instrumentation Error Effects on the Identification Accuracy of Aircraft Parameters," NASA CR-112121, May 1972.

[8]Stepner, D.E., and Mehra, R.K., "Maximum Likelihood Identification and Optimal Input Design for Identifying Aircraft Stability and Control Derivatives," NASA CR-2200, March 1973.

[9]Chen, R.T.N., Eulrich, B.J., and Lebacqz, J.V., "Development of Advanced Techniques for the Identification of V/STOL Aircraft Stability and Control Parameters," CAL Report No. BM-2820-F-1, Aug. 1971.

[10]Holmes, J.E., "Limitations in the Acquisition of Nonlinear Aerodynamic Coefficients from Free Oscillation Data by Means of the Chapman-Kirk Technique," presented at the *AIAA Guidance and Control Conference*, Aug. 1972.

[11]Ross, A.J., "Investigation of Nonlinear Motion Experience on a Slender-Wing Research Aircraft," *Journal of Aircraft*, Vol. 9, Sept. 1972, pp. 625-631.

[12]NASA Symposium on *Parameter Estimation Techniques and Applications in Aircraft Flight Testing*, Edwards Flight Research Center, April 24-25, 1973, NASA TN D-7647, April 1974.

[13]Astrom, K.J. and Eykhoff, P., "System Identification – A Survey," *Automatica*, Vol. 7, Pergamon Press, New York 1971, pp. 123 – 162.

[14]Wolowicz, C.J., Iliff, K.W., and Gilyard, G.B., "Flight Test Experience in Aircraft Parameter Identification," presented at the *AGARD Symposium on Stability and Control,*, Braunschweig, Germany, April 1972.

[15]DeFranco, D., "In-Flight Parameter Identification by the Equation-of-Motion Technique--Application on the Variable Stability T-33 Airplane," Cornell Aeronautical Laboratory Report No. TC-1921-F-3, Dec. 1965.

[16]Gerlach, O.H., "Determination of Performance and Stability Parameters from Non-Steady Flight Test Maneuvers," *SAE National Business Aircraft Meeting*, Wichita, Kansas, March 1970.

[17]Taylor, L., et al., "A Comparison of Newton-Raphson and Other Methods for Determining Stability Derivatives from Flight Data," *Third Technical Workshop on Dynamic Stability Problems*, Ames Research Center, 1968; also presented at the *AIAA Third Flight Test, Simulation and Support Conferences,* Houston, Texas, March 1969.

[18]Bellman, R., et al., "Quasilinearization, System Identification, and Prediction," RAND Corporation, RM-3812, Aug. 1963.

[19]Young, P.C., "Process Parameter Estimation and Adaptive Control," in *Theory of Self-Adaptive Control Systems*, P. Hammond, ed., Plenum Press, New York, 1966.

[20]Tyler, J.S., Powell, J.D., and Mehra, R.K., "The Use of Smoothing and Other Advanced Techniques for VTOL Aircraft Parameter Identification," Final Report to Cornell Aeronautical Laboratory under Naval Air Systems Command Contract No. N00019-69-C 0534, June 1970.

[21]Astrom, K.J., and Wenmark, S., "Numerical Identification of Stationary Time Series," *Sixth International Instruments and Measurements Congress*, Stockholm, Sept. 1964.

[22]Kashyap, R.L., "Maximum Likelihood Identification of Stochastic Linear Dynamic Systems," *IEEE Transactions on Automatic Control,* Feb. 1970.

[23]Gupta, N.K. and Hall, W.E., "Input Design for Identification of Aircraft Stability and Control Coefficients," NASA CR-2493, Feb. 1975.

[24]Mehra, R.K. and Gupta, N.K., "Status of Input Design for Aircraft Parameter Identification," presented at *AGARD Specialist's Conference on Methods for Aircraft State and Parameter Estimation*NASA – Langley Research Center, Hampton, Va., Nov. 1974.

[25]Akaike, H., "Statistical Predictor Identification," *Annals of the Institute of Statistics* Vol. 22, 1970.

[26]Kendall, M.G., and Stuart, A., *The Advanced Theory of Statistics,* Vol. I and II, Charles Griffin, London, 1967.